Conservation of Momentum

Momentum is a **vector quantity** which depends on both the **mass** and the **velocity** of an object. The **total momentum** is always conserved in a collision. This is a fact that can be used to predict velocities both before and after a collision, or predict the **recoil velocity** of a gun or cannon.

We will consider three different types of collision below which will all demonstrate the principle of **conservation of momentum**.

Collide & Lock Collisions

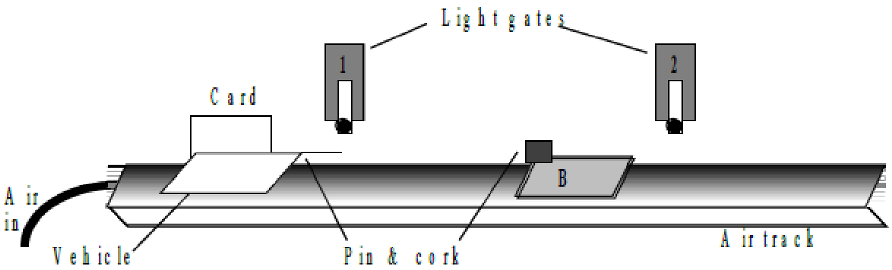
A **collide-and-lock** collision is one where the objects are moving separately before the collision then stick together and move off as one after the collision. An example of this would be a car accident where both cars stick together.

Experiment – Collide & Lock Collisions

This experiment will investigate the momentum before and after a collision where the objects stick together and move off as one.

You will need:

- Linear air track
- Two carts for the air track
- Various masses
- Cork
- Needle
- Light gates
- Timers



This experiment can also be carried out using wheeled dynamics carts – rails are ideal for ensuring the vehicles move only in **one dimension**.

Instructions

1. Set up the experiment as shown in the diagram above – use a piece of cork on one cart and a needle on the second card. Ensure that the light gates are connected to timers and set to either measure the time interval or calculate the speed of the object directly.

2. Fit a mask of known size to both of the carts – check that it breaks the beam of the light gate and that this signal is successfully sent to the timing device.

3. Construct a table similar to the one below for noting your results:

Mass of Vehicle A (kg)	Mass of Vehicle B (kg)	Velocity of Vehicle A Before	Velocity of Vehicle B Before	Total Momentum Before (kgm/s)	Velocity of Vehicles A & B After	Total Momentum After (kgm/s)

4. Keep cart B stationary, and push carts A towards it – let both carts collide and move off together.

5. Use the first light gate to measure the speed of cart A **before** the collision.

6. Use the second light gate to measure the speed of both of the carts **after** the collision.

7. Calculate the **total momentum** before and after the collision – remember the equation for momentum is:
$$p = mv$$

8. Compare the total momentum before the collision to the total momentum after the collision.

9. Repeat with different masses fitted to both carts.

Conservation of Momentum

The **total momentum before** the collision is **equal** to the **total momentum after** the collision – this is known as **conservation of momentum**.

The momentum before the collision is the sum of the individual momenta of the carts – if cart B is stationary, its momentum will be zero. The total momentum will just be the momentum of cart A. The total momentum after the collision will use the **combined mass** of both vehicles. We have:

$$\text{Total Momentum Before} = \text{Total Momentum After}$$
$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

Worked Example

A car of mass 1000 kg travelling at 15 m/s crashes into the back of a second car of mass 800 kg which was travelling at 5 m/s in the same direction. The cars lock together after the collision. What velocity do the cars move off with?

*First of all, calculate the **total momentum before** the collision:*

$$p_{before} = m_1u_1 + m_2u_2$$
$$p_{before} = (1000 \times 15) + (800 \times 5)$$
$$p_{before} = 15{,}000 + 4{,}000$$
$$p_{before} = 19{,}000 kgms^{-1}$$

*Now calculate the **total momentum after** the collision. This time, we have a new mass which is the sum of the masses of the individual cars:*

$$p_{after} = (m_1 + m_2)v$$
$$p_{after} = (1000 + 800)v$$
$$p_{after} = 1{,}800v$$

By Conservation of Momentum,

Total Momentum Before = Total Momentum After

$$19{,}000 = 1{,}800v$$
$$v = \frac{19{,}000}{1{,}800}$$
$$v = 10.56 ms^{-1}$$

Questions

1. Police are investigating a car accident and through analysing the cars and the tyre tracks on the road they have found the following information:
    - Mass of Car A = 700 kg
    - Mass of Car B = 900 kg
    - Velocity of both cars after collision = 8 m/s

   Car B is known to have been stationary before the collision. The cars locked together in the collision.

   a) Calculate the **total** momentum **after** the collision took place.
   b) Find an expression for the total momentum **before** the collision.
   c) Use the principle of **conservation of momentum** to work out the velocity of Car A before the collision.

2. A bullet is fired from a gun and embeds itself in a box of sand which is suspended as a pendulum. The bullet has a mass of 0.05 kg and is travelling with a velocity of 250 m/s. The box of sand has a mass of 3 kg.

   a) Calculate the velocity that the bullet and box move off with after the collision.
   b) Calculate the maximum height to which the bullet and box will swing.

Collide & Split Collisions

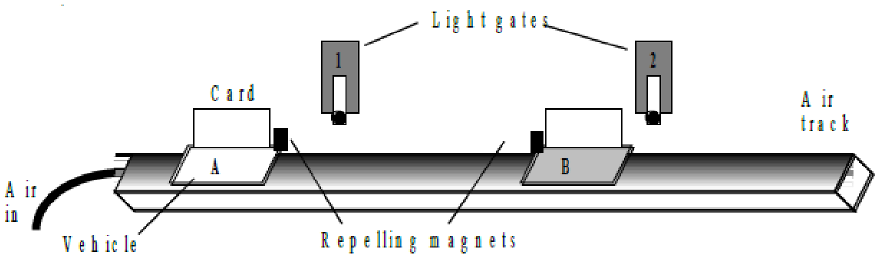
A **collide-and-split** collision is one where two objects collide and then move off separately after the collision. An example of this would be in the game of snooker where the cue ball strikes the object ball and they both move off separately after the collision.

Experiment – Collide and Split Collisions

This experiment will investigate momentum of objects before and after a collision where both vehicles move off separately.

You will need:

- Linear air track
- Two carts for the air track
- Various masses
- Magnets
- Needle
- Light gates
- Timers



This experiment can also be carried out using wheeled dynamics carts – rails are ideal for ensuring the vehicles move only in **one dimension**. In this case, repelling magnets can be used on the carts to ensure that the do not stick together following the collision.

Instructions

1. Set up the experiment as shown in the diagram above – use repelling magnets to ensure that the carts do not stick together. Ensure that the light gates are connected to timers and set to either measure the time interval or calculate the speed of the object directly.

2. Fit a mask of known size to both of the carts – check that it breaks the beam of the light gate and that this signal is successfully sent to the timing device.

3. Construct a table similar to the one below for noting your results:

Mass of Vehicle A (kg)	Mass of Vehicle B (kg)	Velocity of Vehicle A Before	Velocity of Vehicle B Before	Total Momentum Before (kgm/s)	Velocity of Vehicle A After	Velocity of Vehicle B After	Total Momentum After (kgm/s)

4. Keep cart B stationary, and push carts A towards it – let both carts collide and move off separately.

5. Use the first light gate to measure the speed of cart A **before** the collision.

6. Use the second light gate to measure the speed of both of the carts **after** the collision – you must ensure that the timer is set up to make **two** measuremens.

7. Calculate the **total momentum** before and after the collision – remember the equation for momentum is:
$$p = mv$$

8. Compare the total momentum before the collision to the total momentum after the collision.

9. Repeat with different masses fitted to both carts, and with different starting velocities. You can create a head-on collision – remember the velocity of one of the vehicles in this case will be **negative**.

Conservation of Momentum

As before, the **total momentum before** the collision is **equal** to the **total momentum after** the collision. In this case, the total momentum before is the sum of the momenta of the individual carts before, and the total momentum after is the sum of the individual momenta after the collision.

$$\text{Total Momentum Before} = \text{Total Momentum After}$$
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Worked Example

Snooker balls all have an equal mass of 0.16 kg. In a simple shot, the cue ball travels towards a stationary red ball at a speed of 8 m/s. The cue ball collides with the red ball and after the collision the red ball moves off with a speed of 6 m/s. Calculate the speed the cue ball moves off with.

*First of all, calculate the **total momentum before** the collision:*

$$p_{before} = m_1u_1 + m_2u_2$$
$$p_{before} = (0.16 \times 8) + (0.16 \times 0)$$
$$p_{before} = 1.28 + 0$$
$$p_{before} = 1.28 kgms^{-1}$$

*Now calculate the **total momentum after** the collision:*

$$p_{after} = m_1v_1 + m_2v_2$$
$$p_{after} = (0.16 \times v) + (0.16 \times 6)$$
$$p_{after} = 0.16v + 0.96$$

By Conservation of Momentum,

Total Momentum Before = Total Momentum After

$$1.28 = 0.16v + 0.96$$
$$0.32 = 0.16v$$
$$v = \frac{0.32}{0.16}$$
$$v = 2ms^{-1}$$

Questions

1. In a game of lawn bowls, the bowl of mass 2 kg collides with the stationary jack of mass 0.4 kg. The bowl is moving with a velocity of 12 m/s before the collision. After the collision, the jack moves off with a speed of 20 m/s.

   a) Calculate the total momentum of the bowl and the jack **before** the collision.
   b) Find an expression for the total momentum of the bowl and the jack **after** the collision.
   c) Use the principle of **conservation of momentum** to calculate the velocity of the bowl after the collision.

2. In a car accident, two cars are travelling towards each other. They collide and lock, and the wreckage moves off together. Car A has a mass of 1200 kg and was travelling at 26 m/s before the collision. Car B has a mass of 1700 kg and was travelling **in the opposite direction** with a velocity of 19 m/s. Find the resultant velocity of the wreckage.

3. A train carriage with a mass of 4000 kg breaks free and runs down a hill and collides with a stationary engine of mass 9000 kg at the bottom. Just before the collision, the carriage is moving with a velocity of 40 m/s. After the collision the carriage continues forward with a velocity of 10 m/s. What is the velocity of the train engine after the collision?

Explosions

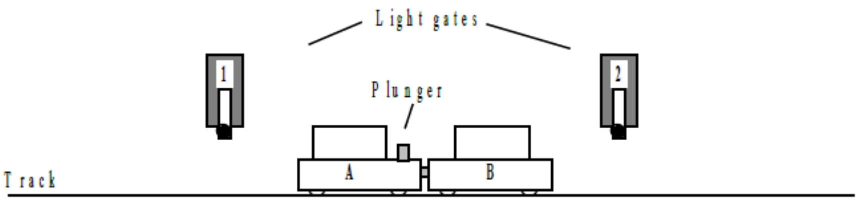
An **explosion** is a type of collision where the initial velocity is zero and then after the collision objects move off in separate directions.

Experiment – Explosions

This experiment will investigate the momentum before and after an explosion.

You will need:

- Linear air track
- Two carts for the air track
- Various masses
- Light gates
- Timers



This experiment can also be carried out using wheeled dynamics carts – rails are ideal for ensuring the vehicles move only in **one dimension**.

Instructions

1. Set up the experiment as shown in the diagram above – use carts with plungers to create the explosion. Ensure that the light gates are connected to timers and set to either measure the time interval or calculate the speed of the object directly.

2. Fit a mask of known size to both of the carts – check that it breaks the beam of the light gate and that this signal is successfully sent to the timing device.

3. Construct a table similar to the one below for noting your results:

Mass of Vehicle A (kg)	Mass of Vehicle B (kg)	Total Momentum Before (kgm/s)	Velocity of Vehicles A After	Velocity of Vehicle B After	Total Momentum After (kgm/s)

4. Place both carts next to each other and ensure they are stationary. Then depress the plunger to create the explosion.

5. Use the first light gate to measure the speed of cart A **after** the collision.

6. Use the second light gate to measure the speed of cart B **after** the collision.

7. Calculate the **total momentum** after the collision – remember the equation for momentum is:

$$p = mv$$

8. Compare the total momentum before the collision to the total momentum after the collision.

9. Repeat with different masses fitted to both carts.

Conservation of Momentum

As with both previous collision types, momentum is conserved for explosions too. Before the explosion, the **total momentum** is **zero**, which is the case after the collision also. The momentum of one object is compensated for by the momentum of the other – this means that one velocity will be positive and the other will be negative.

$$\text{Total Momentum Before} = \text{Total Momentum After}$$
$$0 = m_1 v_1 + m_2 v_2$$

Worked Example

A cannon has a mass of 1500 kg. It fires a cannon ball of mass 10 kg with a velocity of 100 m/s. With what velocity does the cannon move after the cannon ball is fired?

*In this example, we know that the **total momentum before** is **zero** because both the cannon and the cannon ball are stationary:*

$$p_{before} = 0 kgms^{-1}$$

*Now calculate the **total momentum after** the collision:*

$$p_{after} = m_1 v_1 + m_2 v_2$$
$$p_{after} = (100 \times 10) + (1500\, v)$$
$$p_{after} = 1000 + 1500\, v$$

By Conservation of Momentum,

Total Momentum Before = Total Momentum After

$$0 = 1000 + 1500v$$
$$-1000 = 1500v$$
$$v = \frac{-1000}{1500}$$
$$v = -0.67 ms^{-1}$$

*The velocity of the cannon is **negative**. This means that, as expected, the cannon is moving in the opposite direction to the cannon ball. This velocity is known as the **recoil velocity**.*

Questions

1. A cannon has a mass of 900 kg and it fires a cannon ball with a mass of 2 kg. The cannon ball is fired with a velocity of 150 m/s.
   a) What is the total momentum **before** the explosion?
   b) Write down an expression for the total momentum **after** the collision.
   c) Use the principle of **conservation of momentum** to calculate the recoil velocity of the cannon.

2. A cannon with a mass of 1200 kg is used to fire a cannon ball with a mass of 5 kg. The recoil velocity of the cannon is found to be 2 m/s. Use the principle of conservation of momentum to find the velocity that the cannon ball is fired with.

Questions

1. In a game of snooker, the cue ball of mass 0.16 kg strikes the blue ball which has the same mass. The velocity of the cue ball before the collision is 6 m/s, and after the collision the cue ball rebounds in the opposite direction with a velocity of -3 m/s. Calculate the velocity of the blue ball immediately after the collision.

2. Two cars are involved in a collision. Before the collision, Car A (mass = 650 kg) is travelling with a velocity of 20 m/s and Car B is stationary. After the collision, Car A moves with a velocity of 8 m/s and Car B with a velocity of 6 m/s. Use the principle of **conservation of momentum** to find the **mass** of Car B.

3. During an investigation into collisions in the lab, a trolley of mass 0.5 kg is made to collide with a trolley of mass 1 kg. The velocity of the 0.5 kg trolley before the collision is 8 m/s and after the collision it rebounds with a velocity of -5 m/s. If the 1 kg trolley was stationary before the collision, calculate its velocity after the collision.

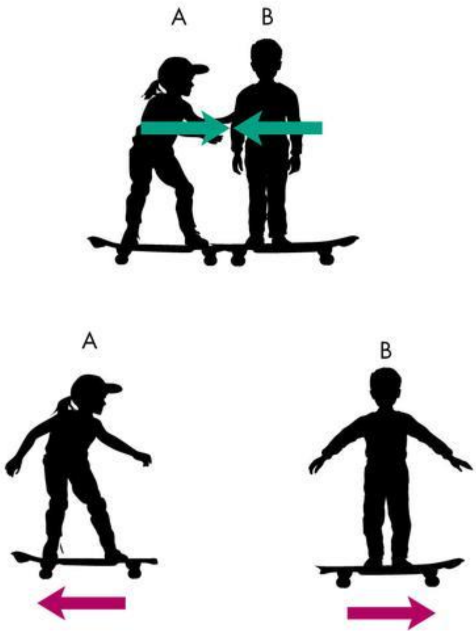
4. In space two spaceships make a docking manoeuvre (joining together). One spaceship has a mass of 1500 kg and is moving at 8 m/s. The second spaceship has a mass of 2000 kg and is approaching from behind at 9 m/s. Determine their common velocity after docking.

5. A firework is launched vertically and when it reaches its maximum height it explodes into 2 pieces. One piece has a mass of 200 g and moves off with a speed of 10 m/s. If the other piece has a mass of 120 g what speed does it have?

Equivalence of Newton's 3rd Law and Conservation of Momentum

Newton's Third Law of motion states:

"For every action force there is an equal and opposite reaction force"



By considering the interaction between two skaters, A and B, initially at rest, derive Newton's Third Law from the principle of conservation of momentum.

You may assume that you are considering an **explosion** where the initial momentum is **zero** which means the final momentum must also be zero.