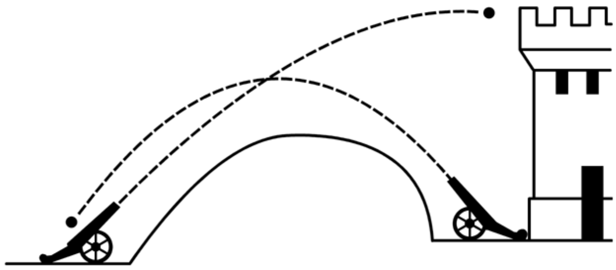
Projectile Motion (2)

So far we have looked at projectiles which are launched **horizontally** – that is, projectiles where the initial vertical velocity is zero. You can also launch projectiles either upwards or downwards, thus giving them an initial vertical velocity. We will consider these types of projectiles here.

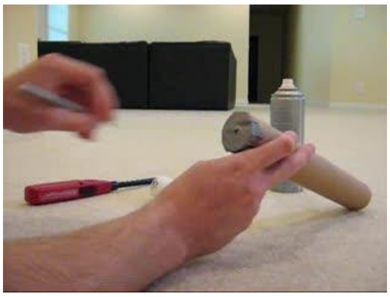


Experiment – The "Ping-Pong" Pistol

A simple projectile launcher can be created from the following material:

- Cardboard tube
- Utility lighter
- Parcel tape
- Ping pong ball
- Aerosol (for example, deodorant spray)

1) Seal one end of the tube with parcel tape. Use a craft knife or a pair of scissors to cut the back of the parcel tape to make a hole big enough to insert the utility lighter into the tube.



2) Spray a **small amount** of the aerosol into the cardboard tube. The aerosol is the fuel for the pistol. However, using too much will starve the pistol of oxygen and it will not ignite.

3) Push the ball down inside the tube. Then carefully push the lighter through the hole in the back of the pistol and pull to light. The ball will be forced out of the pistol by the ignited fuel.

It will be difficult to ensure that the force launching the ball is the same every time. However this simple experiment can be used to see the general effects of launch angle on the range of a projectile.

Compare the range of the projectile when the ball is launched horizontally, at an angle of approximately 45 ° and at an angle close to 90 ° (straight up). Which launch angle gave the greatest range?

Horizontal & Vertical Components

Velocity is a **vector quantity**. We can therefore split this into two **rectangular components**: horizontal and vertical. These velocities can then be analysed separately.

Initial Velocity

Let us consider a "typical projectile" which is launched at a certain angle, θ, above the horizontal as shown in the diagram below. The **initial velocity**, *u*, can be separated into two separate components.

Horizontal Component

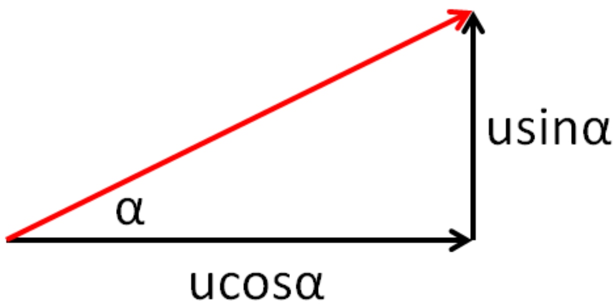
Using trigonometry, we find the horizontal component to be:

$$u_h = u\cos\alpha$$

Vertical Component

Using trigonometry, we find the vertical component to be:

$$u_v = u\sin\alpha$$



Worked Example

A golf ball is launched with an initial velocity of 20 m/s at an angle of 30 ° above the horizontal. Calculate the initial horizontal and vertical components of the motion.

Horizontal:

$$u_h = u\cos\theta$$
$$u_h = 20\cos 30$$
$$u_h = 17.72ms^{-1}$$

Vertical:

$$u_v = u\sin\theta$$
$$u_v = 20\sin 30$$
$$u_v = 10ms^{-1}$$

Horizontal Velocity

From our previous examples, we know that the horizontal velocity of a projectile is **constant**, and obeys the following equation:

$$s = vt$$

Vertical Velocity

The vertical velocity of a projectile is subject to acceleration under gravity of g = -9.8 m/s^2. As this is a constant acceleration, the equations of motion apply:

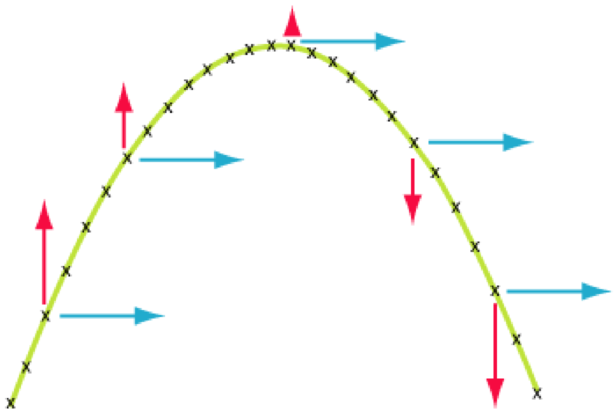
$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

Analysing the Motion of a Projectile

Combining the horizontal and vertical components gives the motion of the projectile shown opposite. The horizontal component remains constant throughout the motion. The vertical component is always accelerating towards the ground – for a projectile launched upwards this means the vertical component slows down to zero and then begin to increase in the downward direction as shown.



Maximum Height

We can see from the diagram above that at the **maximum height** of a projectile the **vertical velocity** is **equal to zero**. Knowing this allows us to work out the time taken to reach the maximum height, or what the maximum height will be for a given launch velocity.

Time Taken

We use the initial vertical velocity, u_v, to find the time take to reach the maximum height:

- $u = u_v$
- $v = 0$
- a = -9.8 m/s^2
- s = X
- t = ?

$$v = u + at$$

Maximum Height

The initial velocity can also be used to work out the maximum height:

- $u = u_v$
- $v = 0$
- a = -9.8 m/s^2
- s = ?
- t = X

$$v^2 = u^2 + 2as$$

Range

The **range**, *R*, of a projectile is the **horizontal distance** travelled between the starting point and the finishing point. It depends on the **horizontal velocity** and the **time of flight**:

$$R = v_h t$$

The **horizontal velocity** is constant, and it worked out using trigonometry as described above.

The **time of flight** can be worked out by considering the **vertical velocity.** For a projectile which takes off and lands at the **same height**, the time of flight is **double** the time taken to reach the maximum height.

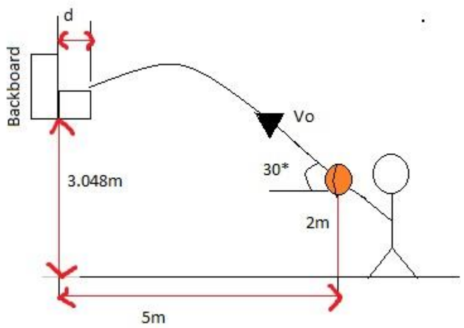
The equations of motion can also be used to work out the time of flight. If the projectile takes off and lands from the same height, the overall vertical displacement is zero:

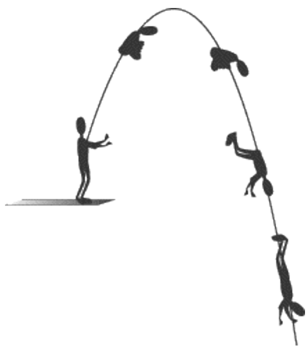
- $u = u_v$
- v = X
- a = -9.8 m/s^2
- $s = 0$
- t = ?

$$s = ut + \frac{1}{2}at^2$$

In this example, you will need to solve a quadratic equation for *t*.

If the projectile lands at a height which is **different** to the take-off height, the overall displacement, *s*, will not be equal to zero. If the projectile lands **above** the take-off point (below left) then the overall displacement is **positive**. If it lands **below** the take-off point (below right) then the overall displacement is **negative**.

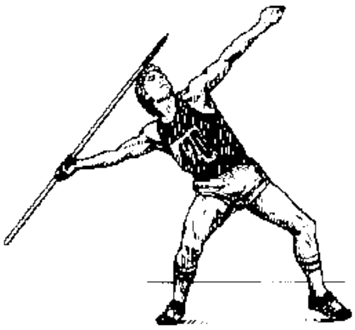




Once you know the time of flight, the equation for range above can be used.

Worked Example

A javelin thrower launches his javelin with an initial velocity of 20 m/s at an angle of 40 ° above the horizontal.



a) Calculate:

i. The initial horizontal velocity
ii. The initial vertical velocity

b) What is the maximum height the javelin will reach?

c) What is the range of the javelin?

a) *Trigonometry is used to find the initial components of the velocity:*

Horizontal:

$$u_h = u\cos\theta$$
$$u_h = 20\cos 40$$
$$u_h = 15.3 m/s$$

Vertical:

$$u_v = u\sin\theta$$
$$u_v = 20\sin 40$$
$$u_v = 12.9 m/s$$

b) *The maximum height of the javelin depends on the* ***vertical velocity*** *so we use the equations of motion:*

- *u = 12.9*
- *v = 0*
- *a = -9.8*
- *s = ?*
- *t = X*

$$v^2 = u^2 + 2as$$
$$0^2 = 12.9^2 + 2(-9.8)s$$
$$0 = 166.4 - 19.6s$$
$$19.6s = 166.4$$
$$s = 8.5m$$

c) *The* ***range*** *of a projectile will depend on the* ***time of flight****. The time of flight is controlled by the* ***vertical velocity****:*

- *u = 12.9*
- *v = X*
- *a = -9.8*
- *s = 0*
- *t = ?*

$$s = ut + \frac{1}{2}at^2$$
$$0 = 12.9t + \frac{1}{2}(-9.8)t^2$$
$$12.9t - 4.9t^2 = 0$$
$$t(12.9 - 4.9t) = 0$$
$$t = 0 \quad ; \quad 4.9t = 12.9$$
$$t = 0 \quad ; \quad t = 2.6s$$

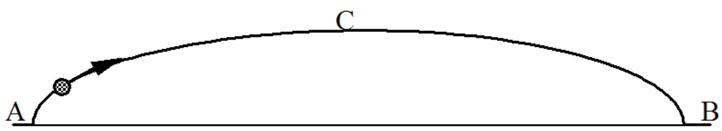
The range of the projectile is then given by:

$$s = vt$$
$$s = 15.3 \times 2.6$$
$$s = 39.8m$$

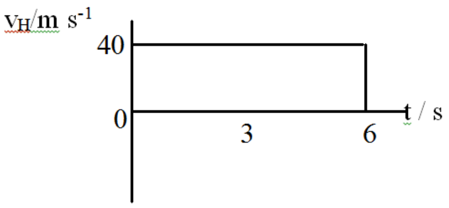
Questions

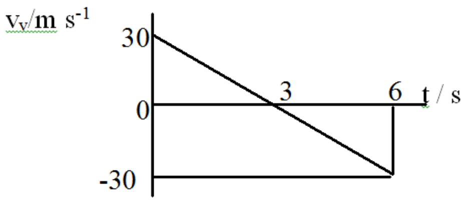
1. A box is released from a plane travelling with a horizontal velocity of 300 ms^{-1} and a height of 300 m, find:
   a) how long it takes the box to hit the ground
   b) the horizontal distance between the point of impact and the release point
   c) the position of the plane relative to the box at the time of impact.

2. A projectile is fired across level ground taking 6 s to travel from A to B. The highest point reached is C. Air resistance is negligible.



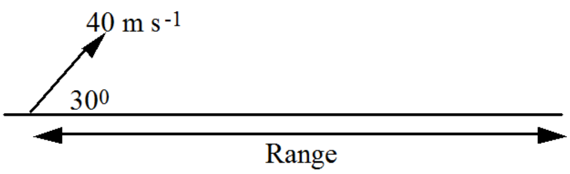
Velocity-time graphs for the horizontal and vertical components of the motion are shown below:





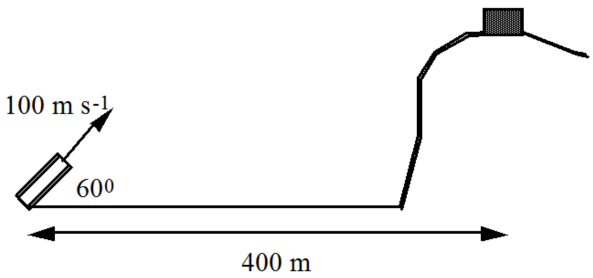
   a) Describe the horizontal and vertical motion of the projectile.
   b) Use a vector diagram to find the resultant velocity (speed and angle) of the projectile at point A.
   c) Find the height above the ground at point C.
   d) Find the range of the projectile from A to B.

3. An object of mass 5 kg is propelled with a speed of 40 ms^{-1} at an angle of 30° to the horizontal.



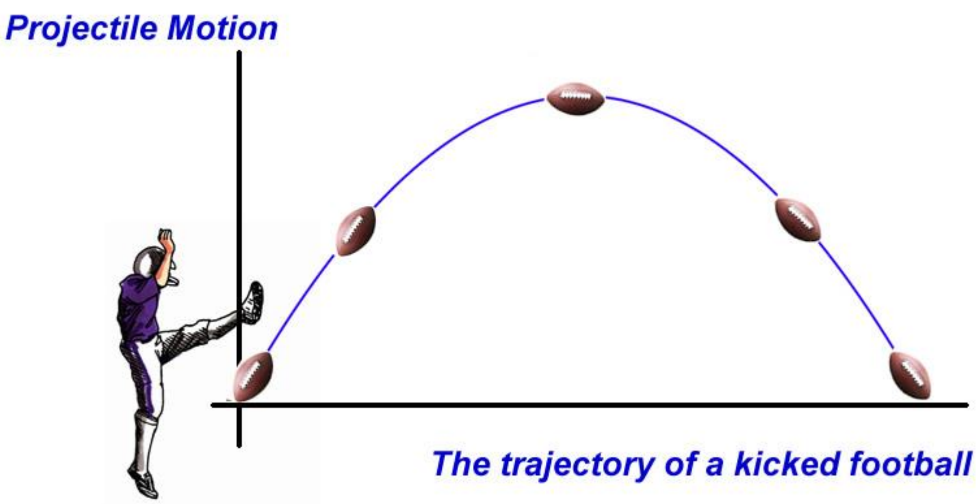
a) Find the initial horizontal and vertical components of the velocity.
b) Calculate the maximum vertical height reached.
c) Find the total time of flight of the projectile.
d) What is the range of the projectile?

4. A missile is launched at 60° to the ground and strikes a target on a hill as shown below. The initial speed of the projectile is 100 ms^{-1}.

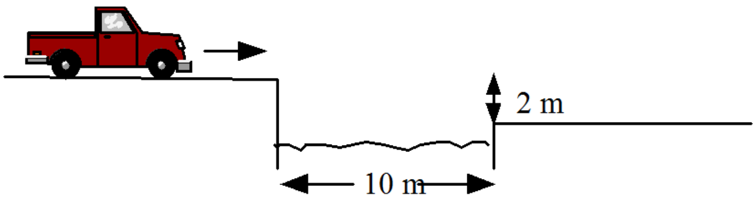


a) Find the time taken to reach the target.
b) Calculate the height of the target above the ground.

5. A football is kicked with an initial velocity of 20 m/s at an angle of 30 ° above the horizontal. A set of goal posts are 15 m away from where the ball is kicked. The goal posts are 7 m high. Does the football go over the goal posts? You must justify your answer by calculation.



6. A stunt driver hopes to jump across a canal of width 10 m. The drop to the other side is 2 m as shown.

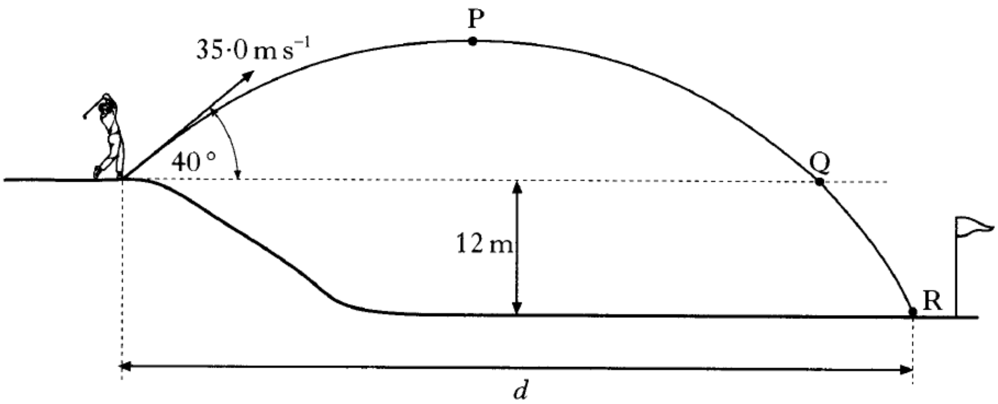


a) Calculate the horizontal speed required to reach the other side of the canal.
b) State any assumptions you have made.

7. A golfer on an elevated tee hits a golf ball with an initial velocity of 35.0 m/s at an angle of 40 ° to the horizontal.

The ball travels through the air and hits the ground at point R.

Point R is 12 m below the height of the tee as shown.



a) Calculate:
   i. The initial horizontal component of the velocity.
   ii. The initial vertical component of the velocity.
   iii. The time taken for the ball to reach its maximum height at P.

b) From its maximum height at point P, the ball falls to point Q, which is at the same height as the tee. It then takes a further 0.48 s to travel from Q until it hits the ground at R. Calculate the total horizontal distance travelled by the ball.