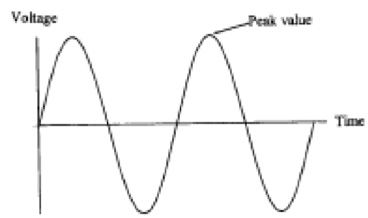
CfE Higher – Electricity – Summary Notes

Monitoring and Measuring Alternating Current

An a.c. supply produces a flow of charge in a circuit that regularly reverses direction. The symbol for an a.c. supply is shown below:-

An a.c. supply would produce a trace that shows alternating peaks and troughs.

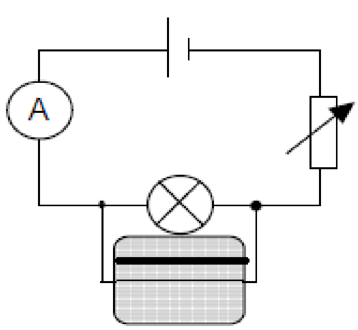


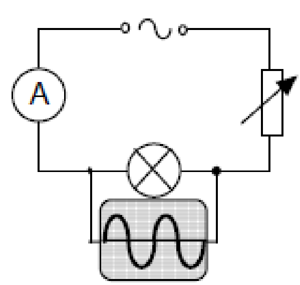
The maximum voltage is called the **peak value**.
From the graph it is obvious that the peak value would not be a very accurate measure of the voltage available from an alternating supply.

In practice the value quoted is the **root mean square** (r.m.s.) voltage.

The r.m.s. value of an alternating voltage or current is defined as being equal to the value of the direct voltage or current which gives rise to the same heating effect (same power output).

Consider the following two circuits which contain identical lamps.

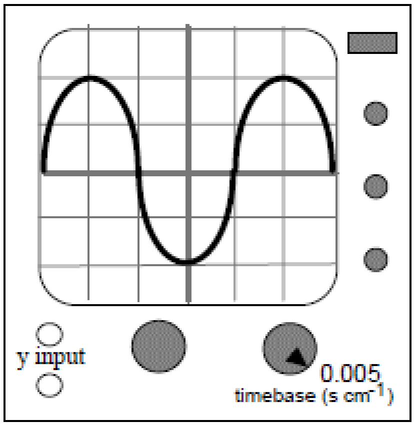




The variable resistors are altered until the lamps are of equal brightness. As a result the direct current has the same value as the effective alternating current (i.e. the lamps have the same power output). Both voltages are measured using an oscilloscope giving the voltage equation below. Also, since V=IR applies to the r.m.s. valves and to the peak values a similar equation for currents can be deduced.

$$V_{r.m.s.} = \frac{1}{\sqrt{2}} V_{peak}$$ and $$I_{r.m.s.} = \frac{1}{\sqrt{2}} I_{peak}$$

An oscilloscope can be used to find the frequency of an a.c. supply as shown below.



Time base = 0.005 s cm^{-1}

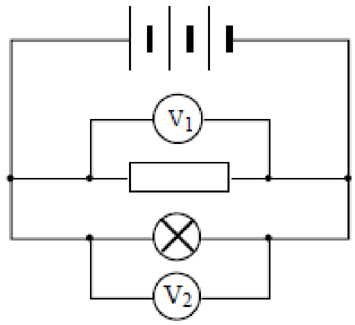
Wavelength = 4 cm

Time to produce one wave = 4 × 0.005
= 0.02 s

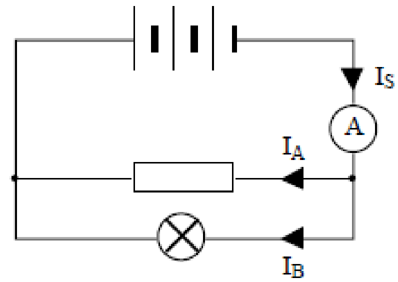
$$\text{Frequency} = \frac{1}{\text{time to produce one wave}}$$

$$= \frac{1}{0.02} \quad = 50 \text{ Hz}$$

Current and Potential Difference or Voltage in Parallel Circuits
The potential difference across components in parallel is the same for all components.
The sum of the currents in parallel branches is equal to the current drawn from the supply.



$V_1 = V_2$



$I_S = I_A + I_B$

Electrical Resistance
Resistance is a measure of the opposition of a circuit component to the flow of charge or current through that component. The greater the resistance of a component, the less will be the current through that component.
All normal circuit components have resistance and the resistance of a component is measured using the relationship

$$R = \frac{V}{I} \quad \text{or} \quad V = I R$$

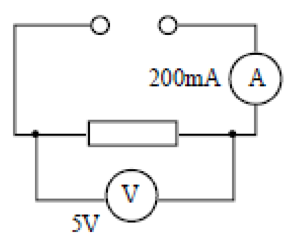
R = resistance
V = potential difference (voltage)
I = current

Resistance is measured in ohms, **Ω**.
Potential difference (or voltage) is measured in volts, **V**.
Current is measured in amperes, **A**.
This relationship is known as **Ohm's Law**, named after a German physicist, Georg Ohm.
For components called **resistors**, the resistance remains approximately constant for different values of current therefore the ratio **V/I** (= R) remains constant for different values of current.

Example
Calculate the resistance of the resistor in the diagram opposite.
Ensure that all quantities are stated in the correct units.

$R = ?$
$V = 5\ V$
$I = 200\ mA = 0.2\ A$

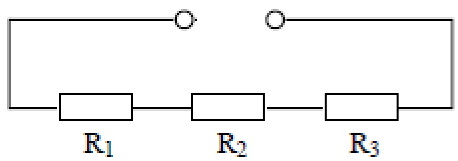
$$R = \frac{V}{I} = \frac{5}{0.2} = 25\ \Omega$$



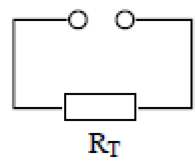
Resistors in Series
When more than one component is connected in series, the **total resistance** of all the components is equivalent to one single resistor, R_T, calculated using the relationship

$$R_T = R_1 + R_2 + R_3$$

For the following circuit with three components in series,



is equivalent to

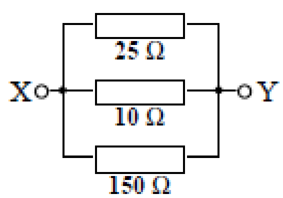


The above relationship is true for two or more components connected in series.

Resistors in Parallel
When more than one component is connected in parallel, the **total resistance** of all the components is equivalent to one single resistor, R_T, calculated using the relationship

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

A typical parallel circuit is shown below.

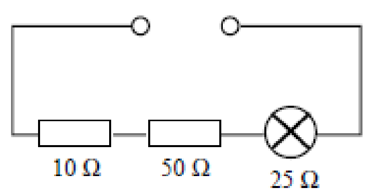


By applying the parallel resistors relationship it can be shown that the total resistance for this circuit is 8.8Ω.

Example 1 Components in series
Calculate the total resistance of the circuit opposite.

$R_1 = 10\ \Omega$
$R_2 = 50\ \Omega$
$R_3 = 25\ \Omega$

$R_T = R_1 + R_2 + R_3$
$R_T = 10 + 50 + 25 = 85\ \Omega$



Example 2 Components in parallel
Calculate the total resistance of the components above when connected in parallel.

$R_1 = 10\ \Omega$
$R_2 = 50\ \Omega$
$R_3 = 25\ \Omega$

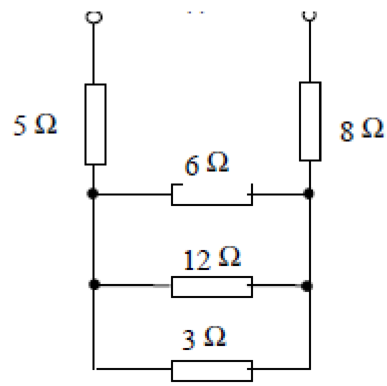
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{10} + \frac{1}{50} + \frac{1}{25}$$

$$= \frac{5}{50} + \frac{1}{50} + \frac{2}{50} = \frac{8}{50}$$

$$\frac{1}{R_T} = \frac{8}{50} \quad \textit{therefore} \quad \frac{R_T}{1} = \frac{50}{8} \qquad R_T = 6.5\ \Omega$$

For components in series, R_T is **always** greater than the largest resistance. For components in parallel, R_T is **always** less than the smallest resistance.

Example 3
Find the total resistance of the following resistor combination.



Firstly, deal with the parallel combination.

$R_T = ?$ $R_1 = 6\Omega$ $R_2 = 12\Omega$ $R_3 = 3\Omega$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$1/ R_T = 1/6 + 1/12 + 1/3$
$1/ R_T = 2/12 + 1/12 + 4/12$
$1/ R_T = 7/12$
$R_T = 1.7\Omega$

Now apply the rule for resistors in series

$$R_T = R_1 + R_2 + R_3$$

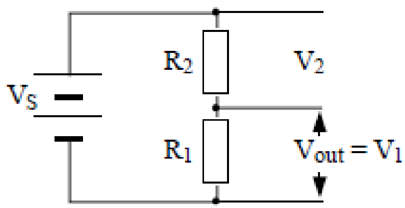
$R_T = 5 + 1.7 + 8$

$R_T = 14.7\Omega$

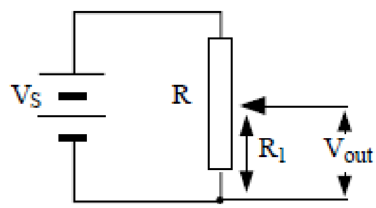
Potential Divider Circuits

A potential divider is a device or a circuit that uses two (or more) resistors or a variable resistor (potentiometer) to provide a fraction of the available voltage (p.d.) from the supply.

Fixed Resistance



Variable Resistance



The p.d. from the supply is divided across the resistors in direct proportion to their individual resistances.

Take the fixed resistance circuit - this is a **series** circuit therefore the current in the same at all points.

$I_{supply} = I_1 = I_2$ where I_1 = current through R_1 and I_2 = current through R_2

Using Ohm's Law:

$$I = \frac{V}{R} \quad \text{hence} \quad \frac{V_S}{R_T} = \frac{V_1}{R_1} = \frac{V_2}{R_2}$$

$$\boxed{V_1 = \frac{R_1}{R_T} \times V_S} \quad \text{or} \quad \boxed{V_1 = \frac{R_1}{R_1 + R_2} \times V_S} \quad \text{and} \quad \boxed{\frac{V_1}{V_2} = \frac{R_1}{R_2}}$$

$R_T = R_1 + R_2$

Example

Calculate the output p.d., V_{out}, from the potential divider circuit shown.

$V_{out} = ?$

$R_1 = 10\ k\Omega$

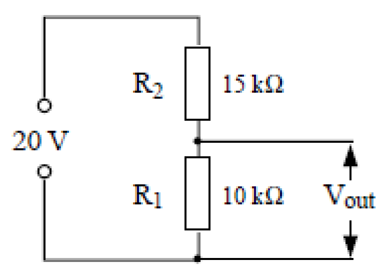
$R_2 = 15\ k\Omega$

$V_S = 20\ V$

$$V_1 = V_{out} = \frac{R_1}{R_1 + R_2} \times V_S$$

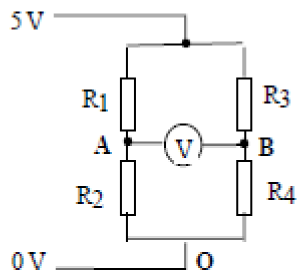
$$= \frac{10}{10 + 15} \times 20$$

$$V_{out} = 8\ V$$



Wheatstone Bridge Circuit

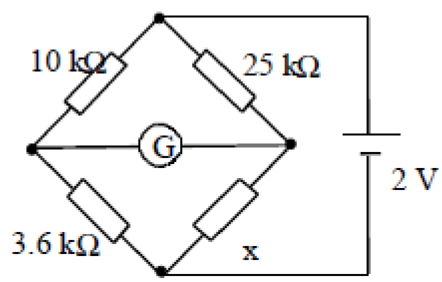
When two potential dividers are connected in parallel as shown below it is known as a Wheatstone Bridge circuit.



The voltmeter measures the potential difference between the two midpoints of the parallel branches. If the voltmeter reads 0V the bridge circuit is said to be balanced. This will happen when the ratios of the resistors in the circuit are equal.

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

This bridge arrangement is often used to find the value of an unknown resistor and can be drawn in a diamond formation.



This Assuming the galvanometer (a sensitive meter) has a zero reading we can apply the above equation.

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

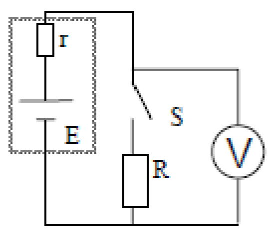
$$10 / 25 = 3.6 / R_x$$

$$R_x = 9.0\text{k}\Omega$$

Electrical Sources and Internal Resistance

When a power supply is part of a closed circuit, it must itself be a conductor. As all conductors have some resistance, the power supply must have its own value of resistance. The name given to the resistance of a power supply is internal resistance and it is given the symbol, r.
A power supply provides the energy for the charges which flow through the closed circuit. The energy given to a coulomb of charge flowing through the power supply is known as the e.m.f. (electromotive force) of the supply. E.m.f. is defined as the electrical potential energy given to each coulomb of charge passing through the supply. E.m.f. is normally measured in volts (V) or joules per coulomb (JC^{-1}) and is given the symbol, E.

A typical circuit diagram which shows e.m.f. and r is shown below.



By applying the principle of conservation of energy, the electrical potential energy available from the supply must be equal the energy "wasted" by the internal resistor added to the energy used by the load (external) resistor, R.

$$E = \text{Voltmeter reading} + \text{Lost volts in } r$$

The voltmeter reading is usually known as the terminal potential difference or t.p.d. and the lost volts can be calculated using Ohm's Law.

$$E = \text{t.p.d.} + Ir$$

The t.p.d. can also be calculated using Ohm's Law.

$$E = IR + Ir$$

This equation is usually written as: -

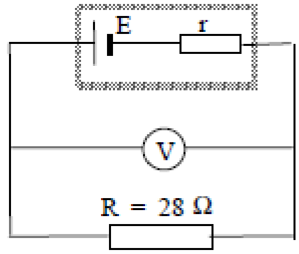
$$E = I(R + r)$$

In a short circuit, which is usually created using a short thick wire, R = 0. This will result in the maximum flow of current and the above equation can be written as: -

$$E = Ir$$

In an open circuit, when no load (external) resistor is connected, no current will be flowing and no energy is wasted. This means that the e.m.f. will be equal to the t.p.d..

Example



A cell of e.m.f. 1.5 V is connected in series with a 28Ω resistor.

A voltmeter measures the voltage across the cell as 1.4 V.

Calculate:

(a) the internal resistance of the cell
(b) the current if the cell terminals are short circuited
(c) the lost volts if the external resistance is increased to 58Ω

(a)

$$E = Ir + IR = Ir + V$$

$$\text{Lost volts} = Ir = E - V = 1.5 - 1.4 = 0.1\ V$$

$$r = \frac{\text{lost volts}}{I} = \frac{0.1}{I}$$

$$I = \frac{V}{R} = \frac{1.4}{28} = 0.05\ A$$

$$r = \frac{0.1}{0.05} = 2\ \Omega$$

(b)

A short circuit occurs when $R = 0$ *(no external resistance)*

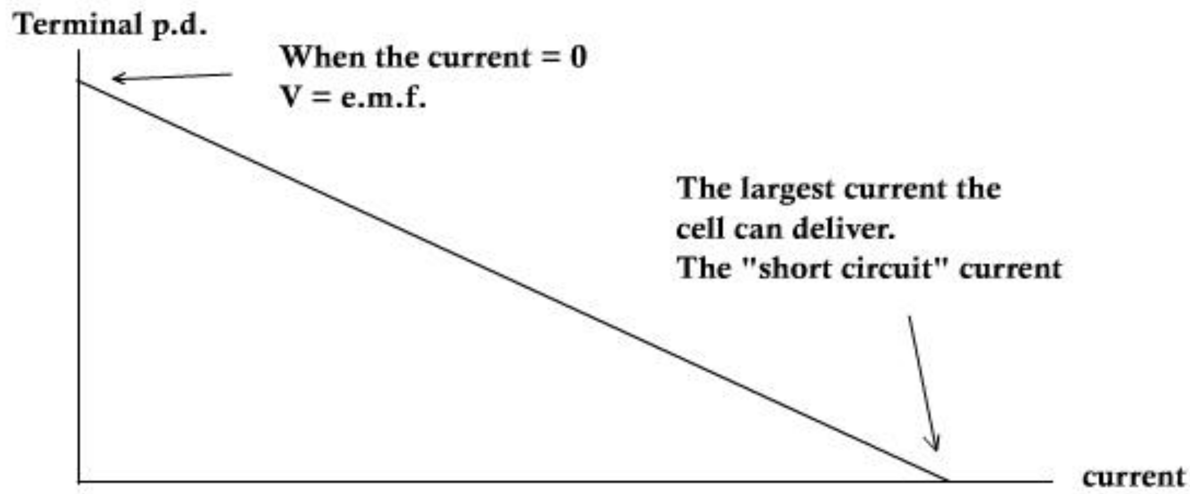
$$E = Ir$$
$$1.5 = I \times 2$$
$$I = 0.75A$$

(c)

$$E = I(R + r)$$
$$1.5 = I(58 + 2)$$
$$1.5 = I(60)$$
$$I = 0.025A$$

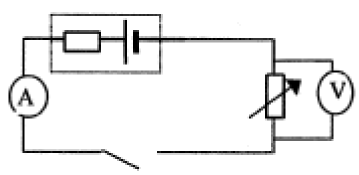
$$\text{Lost volts} = Ir$$
$$\text{Lost volts} = 0.025 \times 2$$
$$\text{Lost volts} = 0.05V$$

When dealing with graphs of t.p.d. vs current it should be noted that the y-intercept is equal to the e.m.f. and that the gradient of the line is equal to the negative of the internal resistance.



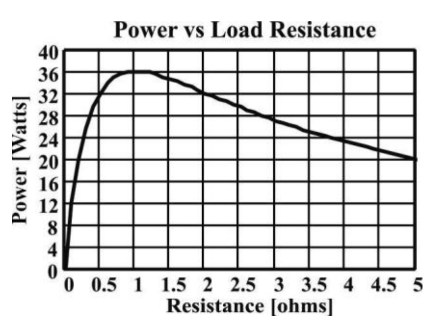
Load Matching Theory

In the following circuit the load is varied by adjusting the variable resistor. The current value is read from the ammeter and the power dissipated in the load is calculated using $P = I^2R$.



The experimental results show that the maximum power transfer takes place when the resistance already present in the circuit is equal to the load resistance. In the above circuit this happens when $r = R$.

Load matching or power transfer theory is often represented by plotting a graph of power vs load resistance. A typical graph is shown below.



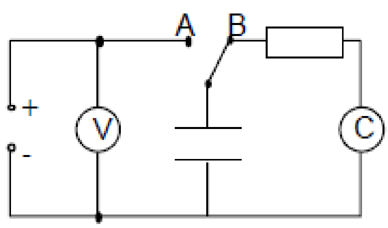
Capacitors

The ability of a component to store charge is known as **capacitance.**
A device designed to store charge is called a **capacitor.**

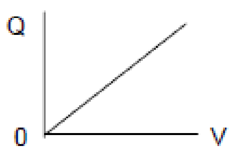
A typical capacitor consists of two conducting layers separated by an insulator.

Circuit symbol ⊣⊢

Relationship between charge and p.d.



The capacitor is charged to a chosen voltage by setting the switch to A. The charge stored can be measured directly by discharging through the coulomb meter with the switch set to B. In this way pairs of readings of voltage and charge are obtained.



Charge is directly proportional to voltage.

$$\frac{Q}{V} = \text{constant}$$

For any capacitor the ratio Q/V is a constant and is called the capacitance.

$$\text{capacitance} = \frac{\text{charge}}{\text{voltage}}$$

farad (F) — capacitance; charge — coulombs (C); voltage — volts (V)

The farad is too large a unit for practical purposes.
In practice the micro farad (μF) = 1×10^{-6} F and the nano farad (nF) = 1×10^{-9} F are used.

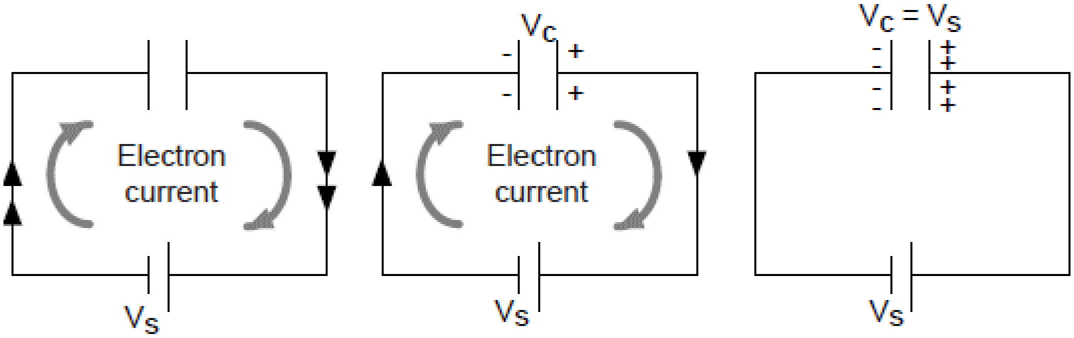
Example
A capacitor stores 4×10^{-4} C of charge when the potential difference across it is 100 V. What is the capacitance ?

$$C = \frac{Q}{V} = \frac{4 \times 10^{-6}}{100}$$

$= 4\ \mu F$

Energy Stored in a Capacitor

A charged capacitor can be used to light a bulb for a short time, therefore the capacitor must contain a store of energy. The charging of a parallel plate capacitor is considered below.

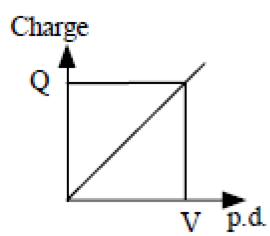


There is an initial surge of electrons from the negative terminal of the cell onto one of the plates (and electrons out of the other plate towards the +ve terminal of the cell).

Once some charge is on the plate it will repel more charge and so the current decreases. In order to further charge the capacitor the electrons must be supplied with enough energy to overcome the potential difference across the plates i.e. work is done in charging the capacitor.

Eventually the current ceases to flow. This is when the p.d. across the plates of the capacitor is equal to the supply voltage.

For a given capacitor the p.d. across the plates is directly proportional to the charge stored Consider a capacitor being charged to a p.d. of V and holding a charge Q.



The energy stored in the capacitor is given by the area under graph

Area under graph $= \frac{1}{2}\, Q \times V$

Energy stored $= \frac{1}{2}\, Q \times V$

If the voltage across the capacitor was constant work done = Q x V, but since V is varying, the work done = area under graph.

Q = C × V and substituting for Q and V in our equation for energy gives:

$$\text{Energy stored in a capacitor} = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

Example

A 40μF capacitor is fully charged using a 50 V supply. How much energy is stored?

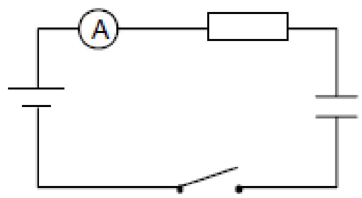
$$Energy = \frac{1}{2}CV^2 = \frac{1}{2} \times 40 \times 10^{6} \times 2500$$

$$= 5 \times 10^{-2}\ J$$

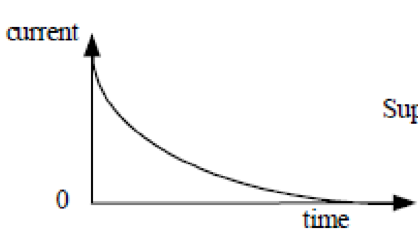
Capacitance in a d.c. Circuit

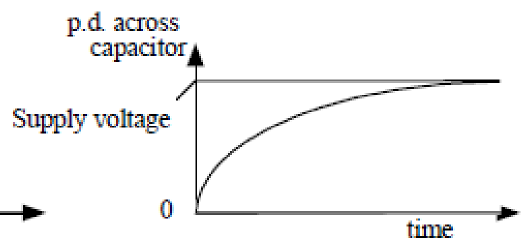
Charging

Consider the following circuit:-

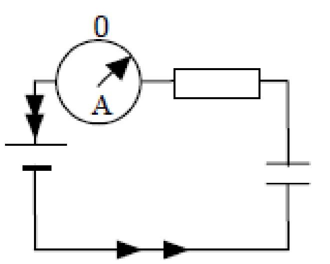


When the switch is closed the current flowing in the circuit and the voltage across the capacitor behave as shown in the graphs below.

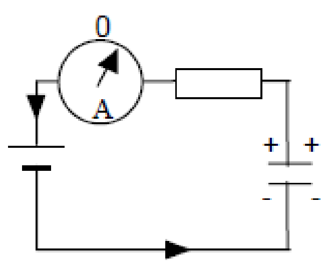




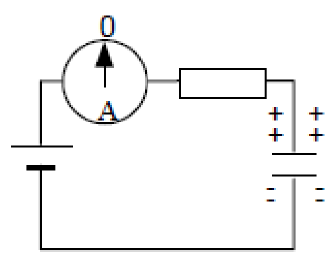
Consider the circuit at three different times.



As soon as the switch is closed there is no charge on the capacitor the current is limited only by the resistance in the circuit and can be found using Ohm's law.



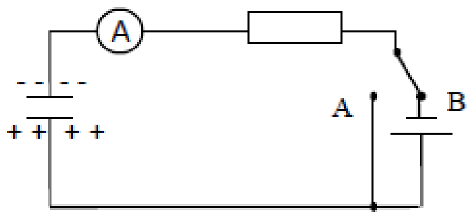
As the capacitor charges a p.d. develops across the plates which opposes the p.d. of the cell as a result the supply current decreases.



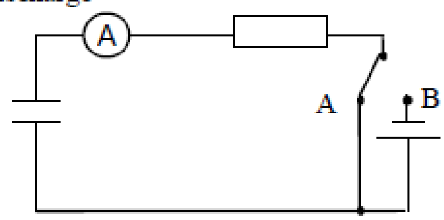
The capacitor becomes fully charged and the p.d. across the plates is equal and opposite to that across the cell and the charging current becomes zero.

Discharging

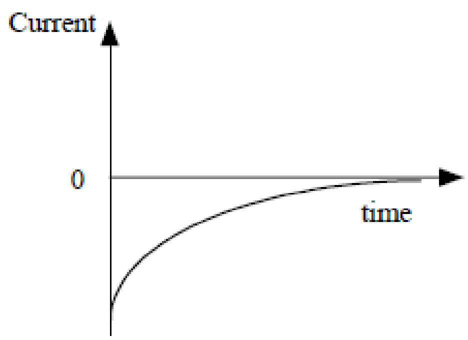
Consider this circuit when the capacitor is fully charged, switch to position B

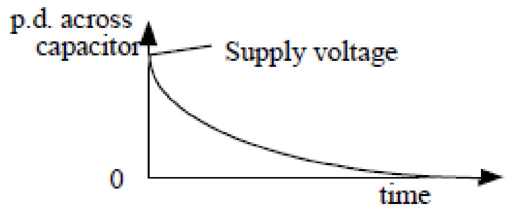


If the cell is taken out of the circuit and the switch is set to A, the capacitor will discharge



While the capacitor is **discharging** the current flowing in the circuit and the voltage across the capacitor behaves as shown in the graphs below.

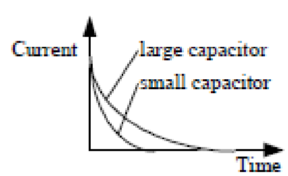


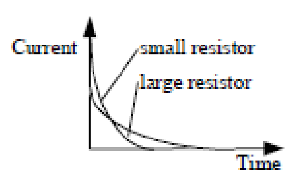


Although the current/time graph has the same shape as that during charging the currents in each case are flowing in opposite directions. The discharging current decreases because the p.d. across the plates decreases as charge leaves them.

Factors affecting the rate of charge/discharge of a capacitor

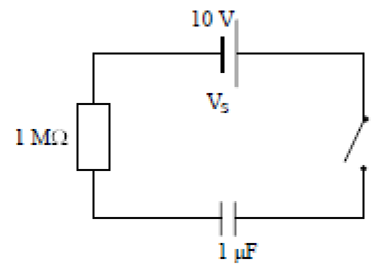
- When a capacitor is charged to a given voltage the time taken depends on the value of the capacitor. The larger the capacitor the longer the charging time, since a larger capacitor requires more charge to raise it to the same p.d. as a smaller capacitor as V= Q
- When a capacitor is charged to a given voltage the time taken depends on the value of the resistance in the circuit. The larger the resistance the smaller the initial charging current, hence the longer it takes to charge the capacitor as Q = It





(The area under this I/t graph = charge. Both curves will have the same area since Q is the same for both.)

Example
The switch in the following circuit is closed at time t = 0



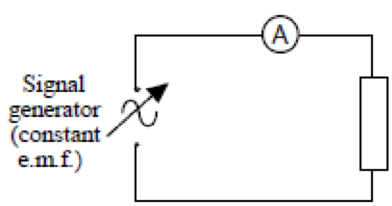
Immediately after closing the switch what is:
(a) *the charge on C*
(b) *the p.d. across C*
(c) *the p.d. across R*
(d) *the current through R.*

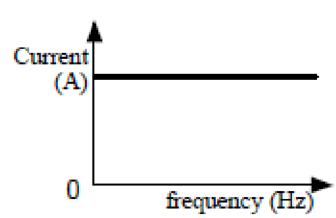
When the capacitor is fully charged what is:
(e) *the p.d. across the capacitor*
(f) *the charge stored.*

(a) *Initial charge on capacitor is zero.*
(b) *Initial p.d . is zero since charge is zero.*
(c) *p.d. is 10 V =* $V_s - V_c = 10 - 0 = 10\ V$
(d) $V = I \times R$
$10 = I \times 1\text{x}10^6$
$I = 1 \times 10^{-5}\text{A}$
(e) 10V
(f) $Q = V \times C$
$Q = 10 \times 1\text{x}10^{-6}$
$Q = 1 \times 10^{-5}\text{C}$

Frequency response of resistor

The following circuit is used to investigate the relationship between current and frequency in a resistive circuit.

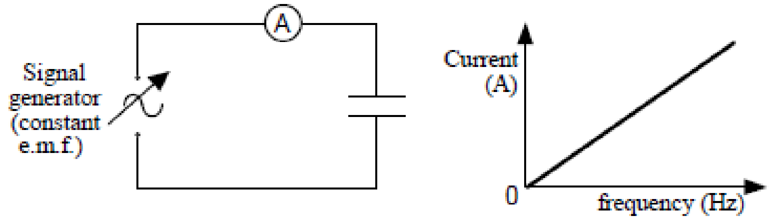




The results show that the current flowing through a resistor is independent of the frequency of the supply.

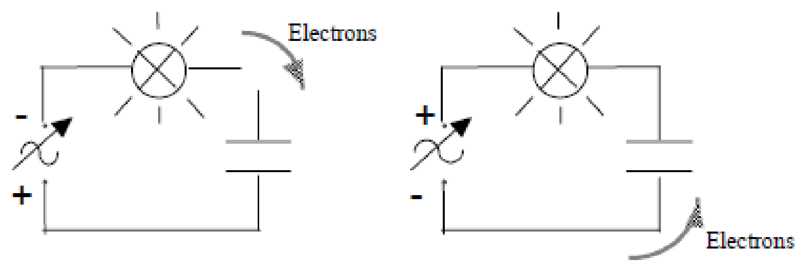
Frequency response of capacitor

The following circuit is used to investigate the relationship between current and frequency in a capacitive circuit.



The results show that the current is directly proportional to the frequency of the supply.

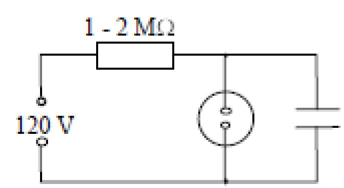
To understand the relationship between the current and frequency consider the two halves of the a.c. cycle.



The electrons move back and forth around the circuit passing through the lamp and charging the capacitor one way and then the other (the electrons do not pass through the capacitor). The higher the frequency the less time there is for charge to build up on the plates of the capacitor and oppose further charges from flowing in the circuit More charge is transferred in one second so the current is larger.

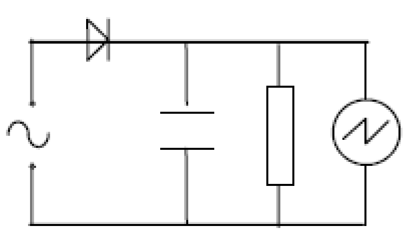
Flashing indicators

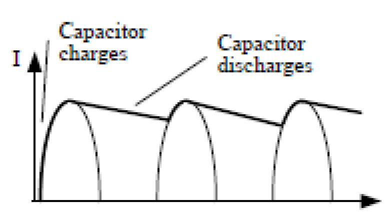
A low value capacitor is charged through a resistor until it acquires sufficient voltage to fire a neon lamp. The neon lamp lights when the p.d. reaches 100 V. The capacitor is quickly discharged and the lamp goes out when the p.d. falls below 80V.



Smoothing

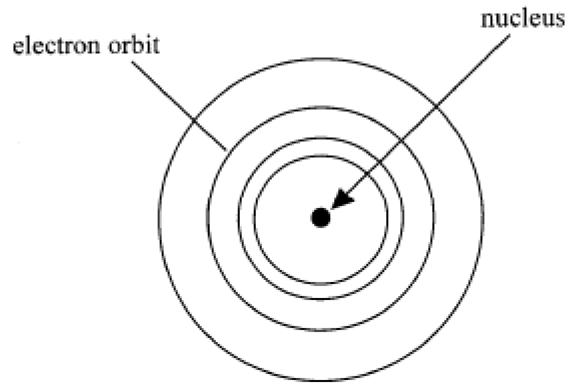
The capacitor in this simple rectifier circuit is storing charge during the half cycle that the diode conducts. This charge is given up during the half cycle that the diode does not conduct. This helps to smooth out the waveform.





Conductors, Insulators and Semiconductors

The basic structure of an atom consists of a central nucleus which is orbited by electrons. These electrons are contained in discrete energy levels. No electrons exist between energy levels.



In insulators the outermost level has no electrons that are free to move. In conductors the outermost level does have electrons that are free to move which allows electrical conduction to take place.

In a solid there will be large numbers of atoms joined together. This means that it is more appropriate to look at energy bands rather than energy levels.

conduction band

band gap

valence band

filled band

INSULATOR BAND DIAGRAM

In insulators,

- the valence band is full
- the conduction band is empty
- the band gap is large
- there is no electrical conduction

(At room temperature)

conduction band

valence band

filled band

CONDUCTOR BAND DIAGRAM

In conductors,

- the valence band is full
- the conduction band contains some electrons
- the band gap is small and can overlap as shown above
- there is electrical conduction

(At room temperature)

A third group of materials called semiconductors exists. The band diagram for a semiconductor is similar to that of a conductor – the band gap between the valence and conduction bands is small enough to allow some electrons to occupy the conduction band.

conduction band

band gap

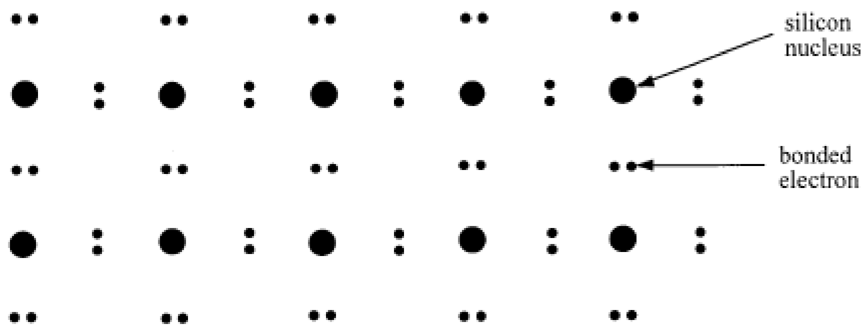
valence band

filled band

SEMICONDUCTOR BAND DIAGRAM

Bonding in semiconductors

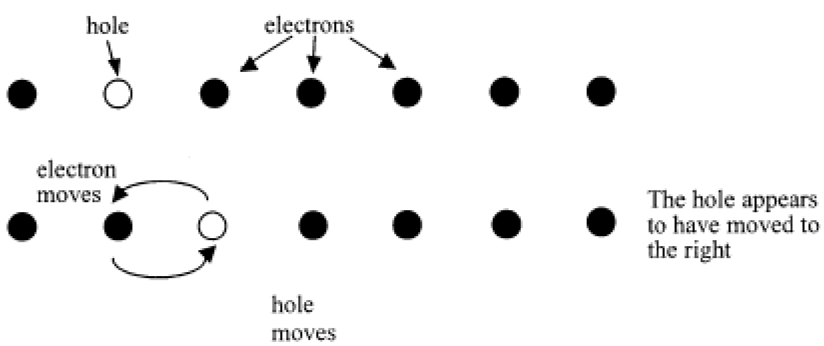
The most commonly used semiconductors are silicon and germanium. Both these materials have a valency of four, that is they have four outer electrons available for bonding. In a pure crystal, each atom is bonded covalently to another four atoms. All of its outer electrons are bonded and therefore there are few free electrons available to conduct. This makes the resistance very large.



Holes

When an electron leaves its position in the crystal lattice, there is a space left behind that is positively charged. This lack of an electron is called a **positive hole**.

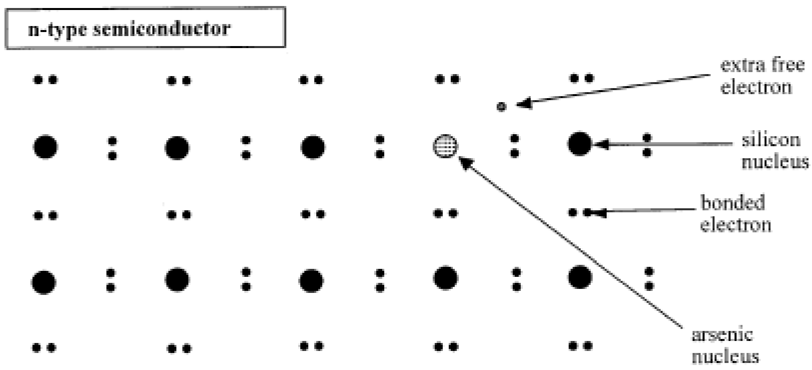
This hole may be filled by an electron from a neighbouring atom, which will in turn leave a hole there. Although it is technically the electron that moves, the effect is the same as if it was the hole that moved through the crystal lattice. The hole can then be thought of as the charge carrier.



In an undoped semiconductor, the number of holes is equal to the number of electrons. Current consists of drifting electrons in one direction and drifting holes in the other.

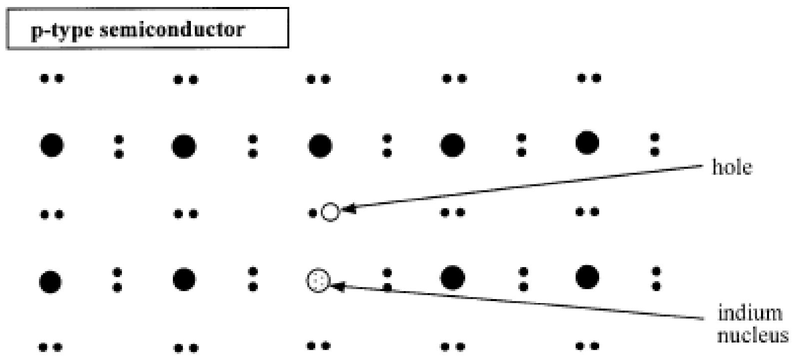
Doping

If an impurity such as arsenic (As), which has **five** outer electrons, is present in the crystal lattice, then four of its electrons will be used in bonding with the silicon. The fifth will be free to move about and conduct. Since the ability of the crystal to conduct is increased, the resistance of the semiconductor is therefore reduced. The addition of an impurity like this is called **doping**.



This type of semiconductor is called **n-type**, since most conduction is by the movement of free electrons, which are, of course, **negatively charged**.

The semiconductor may also be doped with an element like indium (In), which has only three outer electrons. This produces a hole in the crystal lattice, where an electron is 'missing'.



An electron from the next atom can move into the hole created as described previously. Conduction can thus take place by the movement of positive holes. This is called a **p-type** semiconductor, as most conduction takes place by the movement of **positively charged** 'holes'.

Notes on doping

- The doping material cannot simply be added to the semiconductor crystal. It has to be grown into the lattice when the crystal is grown so that it becomes part of the atomic lattice.

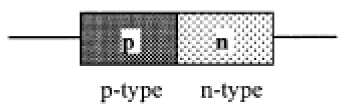
- The quantity of impurity is extremely small; it may be as low as one atom in a million.

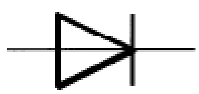
- Although p-type and n-type semiconductors have different charge carriers, they are still both **overall neutral** (just as metal can conduct but is normally neutral).

- Each type of semiconductor will still have small amounts of additional free electrons due to thermal ionisation.

The p-n junction diode

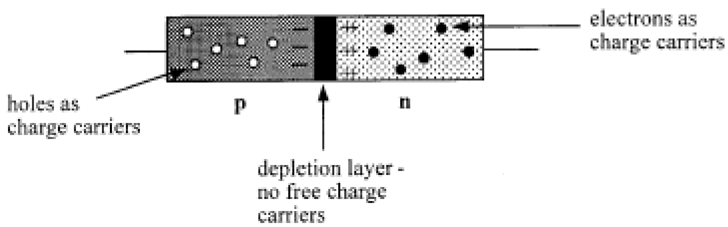
When a semiconductor is grown so that one half is p-type and the other half is n-type, the product is called a **p-n junction diode**.





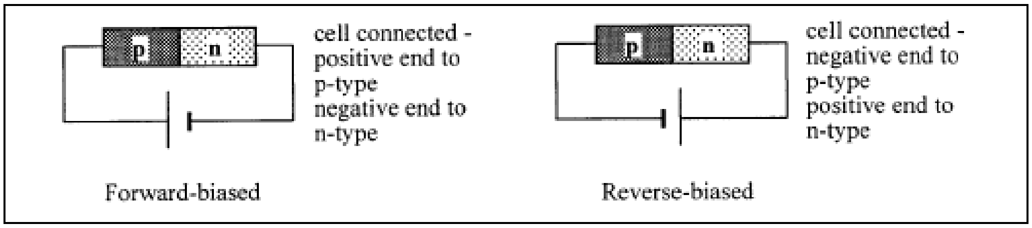
circuit symbol

Some of the free electrons from the n-type material diffuse across the junction and fill some of the holes in the p-type material. This can also be thought of as holes moving in the opposite direction to be filled by electrons. This movement means the area around the junction loses virtually all of its free charge carriers. This area is therefore known as the depletion layer.

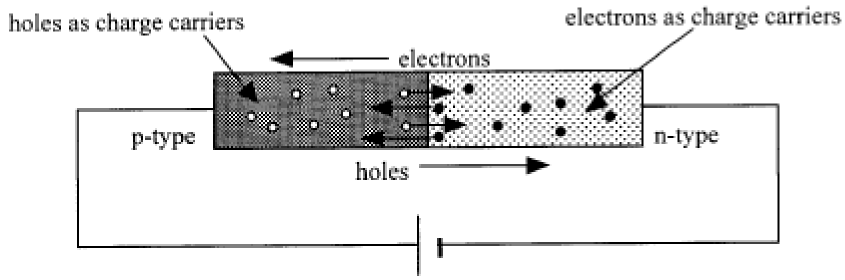


Biasing the diode

To bias a semiconductor device means to apply a voltage to it.
A diode may be biased in two ways.

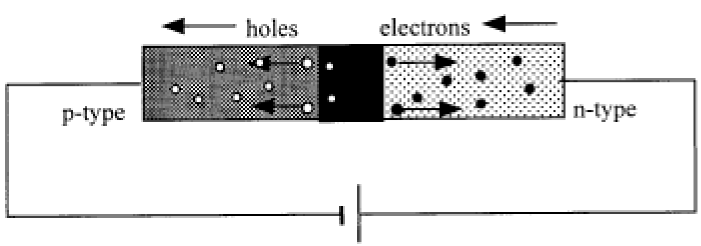


The forward-based diode



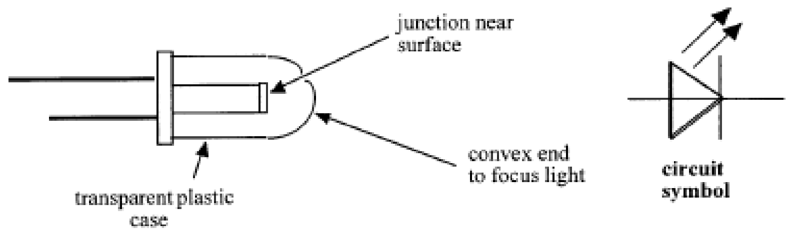
Electrons from the n-type material will be given enough energy from the battery to flow across the junction and round the above circuit in an anti-clockwise direction. This movement will result in a similar movement of holes in the clockwise direction. The diode conducts because the depletion layer has been removed.

The reverse-biased diode



In the reverse biased diode the applied potential difference increases the depletion layer. The diode will become even less likely to conduct.

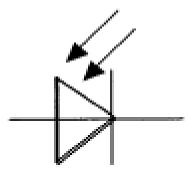
The light-emitting diode



We have seen that in a forward-biased p-n junction diode, holes and electrons pass through the junction in opposite directions. Sometimes holes and electrons will meet and recombine. When this happens, energy is emitted in the form of a photon. For each recombination of electron and hole, one photon of radiation is emitted. In most semiconductors this takes the form of heat, resulting in a temperature rise. In some semiconductors such as gallium arsenic phosphide, however, the energy is emitted as light. If the junction is close to the surface of the material, this light may be able to escape. This makes what we call a **Light Emitting Diode (LED)**. The colour of the light will depend on the materials that are used to make the semiconductor.

The Photodiode

A p-n junction in a transparent coating will react to light. The symbol for a photodiode is shown below.



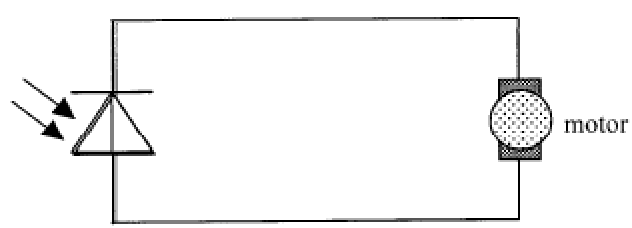
circuit symbol

Photovoltaic mode

In this mode the diode has **no** bias voltage applied. Photons that are incident on the junction have their energy absorbed, freeing electrons and creating electron-hole pairs. A voltage is generated by the separation of the electron and hole. More intense light (more photons) will lead to more electron-hole pairs being produced and therefore a higher voltage. In fact the voltage is proportional to the light intensity.

In this mode, the photodiode will supply power to a load, e.g. a motor. This is the basis of **solar cells**.

Photovoltaic mode circuit



It is interesting to note that in this mode, the photodiode acts like a LED in reverse.