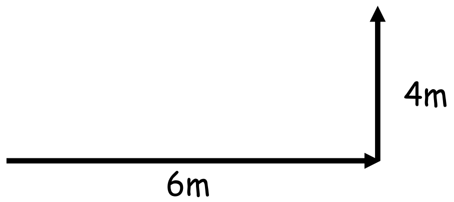
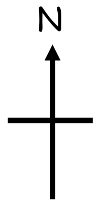
Vector Calculations and Projectile Motion

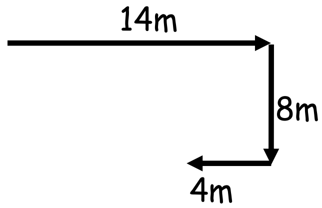
1. Determine the resultant vector in each case

a)





c)



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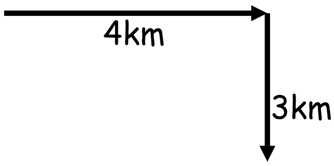
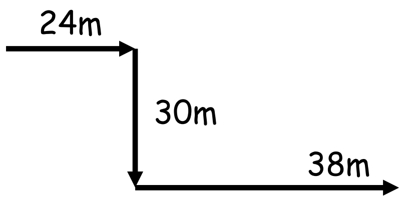
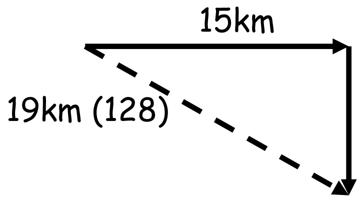


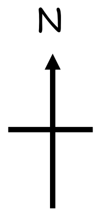
d)



2. Find the unknown vector.

a)





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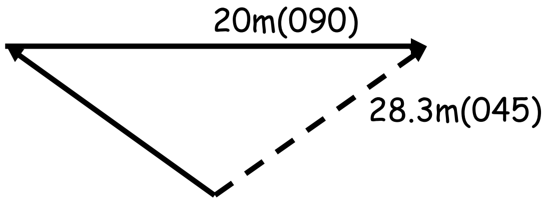

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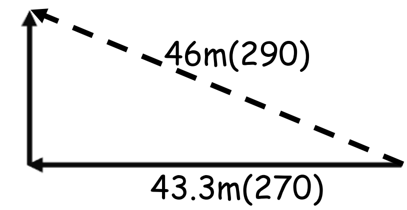

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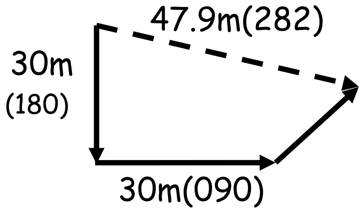
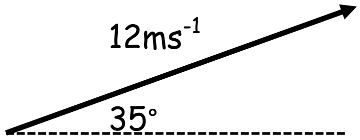


3. Show the Horizontal and Vertical components of the following vectors.

a)



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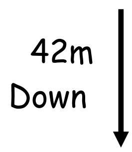

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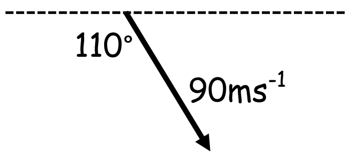
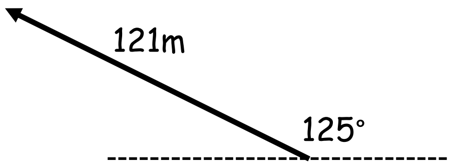
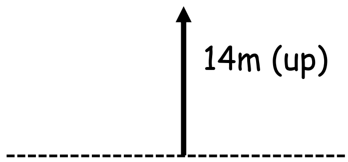


UP
(toward top of page)

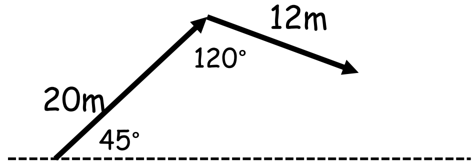
e)



c)



f)



4. Use the process of splitting into Horizontal and Vertical components to determine the RESULTANT vectors below. (Show all workings, sketch a diagram of the path taken)

a) A postie leaves her van to deliver letters. She walks 40m (045) to house A; then 60m (310) to house B; finally, 25m (200) to house C.
Determine the displacement of house C from the van.

b) A model plane flies 70m north; then 70m north west; finally 100m south west before landing. Determine the displacement from the landing point to the take-off position.

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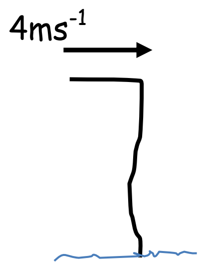
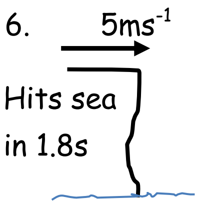
5. A stone is thrown horizontally at $4ms^{-1}$ from the top of a cliff hitting the sea 2.4 seconds later.



a) Describe the path of the stone.
b) How far from the cliff will the stone hit the sea?
c) Calculate the vertical speed of the stone hitting the sea.

Determine the same factors b) & c) shown in Qs.5 for the diagrams below

6.


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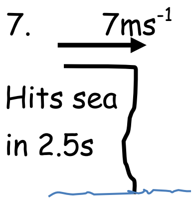

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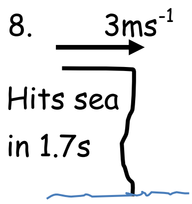

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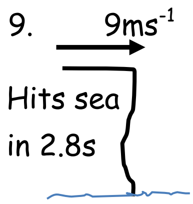

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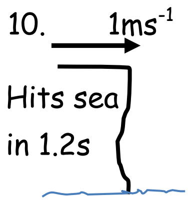
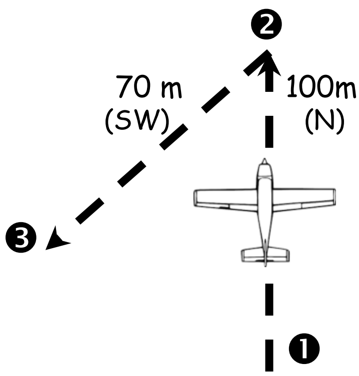


11.



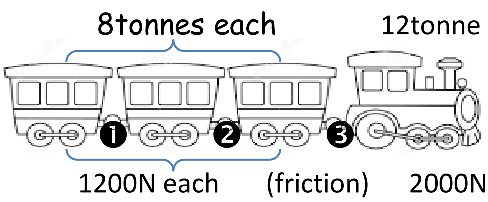
Part of a flight plan, in a model aircraft display, is shown to the left (through check points❶,❷,❸).
This particular 'plane files at $2.5\ ms^{-1}$.
Determine:

a) Displacement❶→❸
b) Minimum time between ❶&❸
c) Average velocity❶→❸

On the actual display day a $1ms^{-1}$ (from) west wind blows.
Determine:

d) How the plane's flight has to change to hit each check point.
e) How flight time ❶→❸ will be affected.

12.



An engine moves a train along a straight track, at a steady speed of $5ms^{-1}$, before accelerating by $1.2ms^{-2}$
Determine, before and during acceleration:

a) The TENSION on coupling ❶, ❷, & ❸
b) The total engine force

ANSWERS

1 a) 7.2m(056) b) 5km(127) c) 12.8m(129) d) 68.9m(116)

2 a) 11.7km(180) b) 15.5m(000) c) 28.3m(315) d) 26.1m(40)

3 a) Hor.= 9.8ms^{-1}(right)
Vert.= 6.9ms^{-1}(up)

b) Hor.= 30.8ms^{-1}(right)
Vert.= 84.6ms^{-1}(down)

c) Hor.= 0m
Vert.= 14m(up)

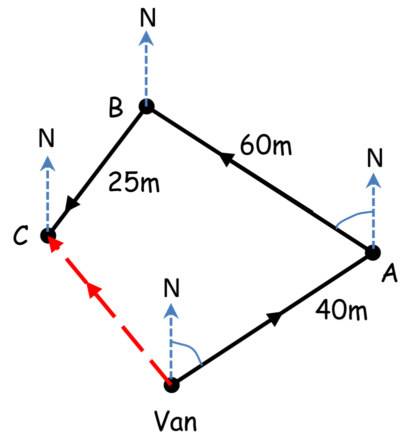
d) Hor.= 0m
Vert.= 42m(down)

e) Hor.= 69m(left)
Vert.= 99m(up)

f) Hor.= 26m(right)
Vert.= 11m(up)

4.

a)

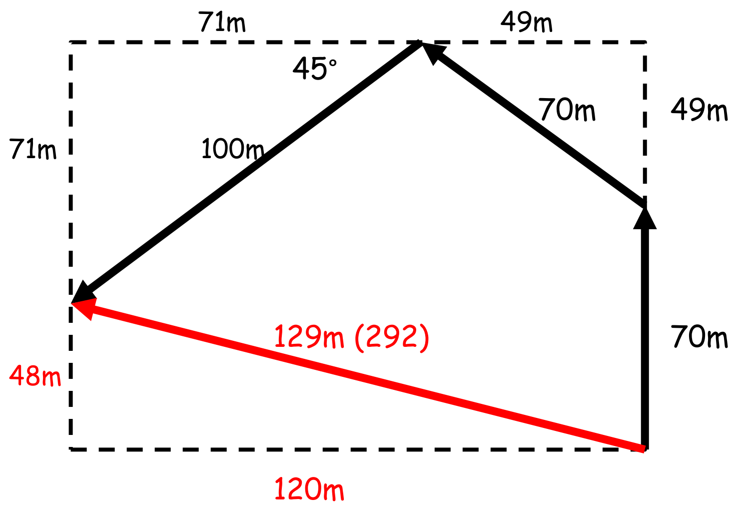


Van→A 28m(north); 28m(east)
A→B 39m(north); 46m(west)
B→C 23m(south); 9m(west)

Resultant Van→C (shown in red) 44m(north); 27m(west) 51.6m(323)

b) Method shown:

- Split each part of flight into horizontal and vertical components (only 2 calculations needed)
- From these determine the horizontal and vertical components of the resultant vector (# in red)
- Use trigonometry to determine bearing; Pythagoras theory to work out the resultant size



5. a)
- Constant horizontal velocity
- Constant vertical acceleration

b)

$$s = \frac{d}{t} \quad \therefore d = s \times t$$
$$d = 4 \times 2.4$$
$$= 9.6m$$

c)

$$a = \frac{v-u}{t}$$
$$\therefore v = a \times t$$
$$= 9.8 \times 2.4$$
$$= 23.5\ ms^{-1}$$

6. b) $s = \frac{d}{t} \;\therefore d = s \times t$

$d = 5 \times 1.8$

$= 9.0m$

c) $a = \frac{v-u}{t}$

$\therefore v = a \times t$

$= 9.8 \times 1.8$

$= 17.6\, ms^{-1}$

7. b) $s = \frac{d}{t} \;\therefore d = s \times t$

$d = 7 \times 2.5$

$= 17.5m$

c) $a = \frac{v-u}{t}$

$\therefore v = a \times t$

$= 9.8 \times 2.5$

$= 24.5\, ms^{-1}$

8. b) $s = \frac{d}{t} \;\therefore d = s \times t$

$d = 3 \times 1.7$

$= 5.1m$

c) $a = \frac{v-u}{t}$

$\therefore v = a \times t$

$= 9.8 \times 1.7$

$= 16.7\, ms^{-1}$

9. b) $s = \frac{d}{t} \;\therefore d = s \times t$

$d = 9 \times 2.8$

$= 25.2m$

c) $a = \frac{v-u}{t}$

$\therefore v = a \times t$

$= 9.8 \times 2.8$

$= 27.4\, ms^{-1}$

10. b) $s = \frac{d}{t} \;\therefore d = s \times t$

$d = 1 \times 1.2$

$= 1.2m$

c) $a = \frac{v-u}{t}$

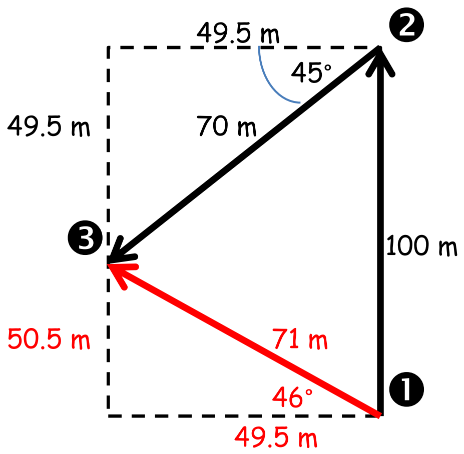
$\therefore v = a \times t$

$= 9.8 \times 1.2$

$= 11.8\, ms^{-1}$

11.

a)



a) Follow method shown in 4.b)

b) $t = \frac{d}{s} = \frac{170}{2.5} = 68\ s$

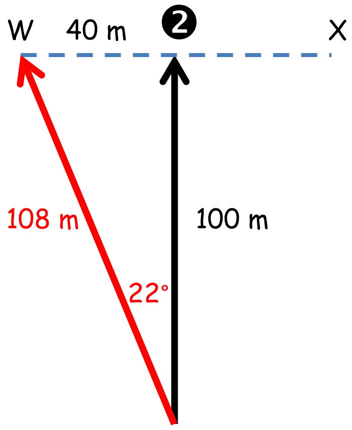
c) $v = \frac{s}{t} = \frac{71}{68} = 1.0\ ms^{-1}$

d)

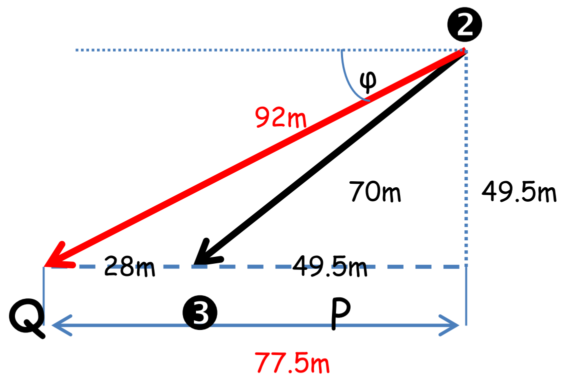
If the model plane tried to fly straight from ❶ to ❷, the wind would carry it toward **X**. To allow for the wind the plane needs to aim for **W**.

❶ to ❷ is 100m, the model would take 40 s to cover this distance, in that time the wind would carry it 40 m off course.

Use trigonometry and Pythagoras theory to work out the resultant vector.



Resultant = 108 m (338)



Flying straight from ❷ to ❸, the wind would carry it toward **P**. To allow for the wind the plane needs to aim for **Q**.

❷ to ❸ is 70m, the model would take 28 s to cover this distance, in that time the wind would carry it 28 m off course.

Angle ❷Y❸ = $\theta = tan^{-1}\ \frac{49.5}{77.5} = 33°$

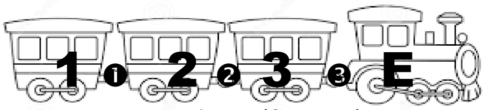
Using the maths principal of Z angles Θ = φ =33° ⇨ bearing = 360 -33 = (227)

Size of resultant $\sqrt{77.5^2 + 49.5^2} = 92\ m$

Resultant ❷→❸ = 92 m (227)

e) The model plane covers the equivalent of 108 m + 92 m through the air ❶→❷→❸
200m at 2.5 ms^{-1} takes 80s, an increase of 12s from the time shown in part b)

12.



a) At a steady speed the tension on the coupling equals the frictional forces it has to overcome.

i. Coupling ❶ drags carriage **1**; T=1200N
ii. Coupling ❷ drags carriage **1**+**2**; T=2400N
iii. Coupling ❸ drags carriage **1**+**2**+**3**; T=3600N

b) **E** has to overcome total friction;

T = 3600 + 2000 = <u>5600N</u>

During acceleration the tension on each coupling has to provide the accelerating force as well as overcome friction.

To accelerate 8 tonnes by 1.2ms^{-2}

$$F = m \times a$$
$$= 8 \times 10^3 \times 1.2 = 9.6kN$$

To accelerate 12 tonnes by 1.2ms^{-2}

$$F = m \times a$$
$$= 12 \times 10^3 \times 1.2 = 14.4kN$$

a)

iv. Coupling ❶ accelerates carriage **1**;
T=1200+9600 = 10800N (10.8kN)

v. Coupling ❷ accelerates carriage **1**+**2**;
T=2400+(9600)x2 = 21600N (21.6kN)

vi. Coupling ❸ accelerates carriage **1**+**2**+**3**;
T=3600+(9600)x3 = 32400N (32.4kN)

b) **E** has to accelerate the whole train;
T=5600+(9600)x3+14400 = 48800N (48.8kN)